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Gauge Theory of Dislocations

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In this continuation of work by the author the notion of the distortion of an ideal crystal structure is generalized and the gauge field is defined, fundamental states ("vacuum configurations") of which are the crystal structure elementary distortions due to dislocations. The form of the structure equations of the connection form defined by this gauge field is discussed.

1. INTRODUCTION

In mechanics of continua, a body is understood as a three-dimensional differentiable manifold $\mathfrak{B}$ having a diffeomorphic mapping

$$\kappa: \quad \mathfrak{B} \to E \tag{1}$$

onto a connected subset of the three-dimensional Euclidean point space E. In this paper we shall additionally assume that $\mathfrak{B}$ is a simply-connected differentiable manifold of class C^∞, diffeomorphic to an open set in E; the latter assumption means that we shall not consider the problems concerning the boundary of the body.

In the case of a crystalline body whose crystal structure is a three-dimensional monoatomic Bravais lattice, the material structure of the body can be described by assigning to each point $p \in \mathfrak{B}$ a triad of base vectors of the lattice:

$$E_{T(p)} = (\underline{E}_a(p);\, a = 1, 2, 3), \qquad \underline{E}_a(p) \in T(S_p) \subset T_p(\mathfrak{B}) \tag{2}$$

where $T(S_p)$ denotes the lattice group of a Bravais lattice $S_p \subset T_p(\mathfrak{B})$, and $T_p(\mathfrak{B})$ is the space tangent to $\mathfrak{B}$ at the point $p \in \mathfrak{B}$ (Trzęsowski, 1987, Section 1).

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If the crystal structure of the body is ideal (i.e., without crystal lattice defects), then there exists a mapping (1) such that

$$\forall p \in \mathfrak{B} | \quad \kappa_{*p}(\underline{E}_a(p)) = \overset{o}{\underline{E}}_a \in T(S_0) \subset \mathbf{E} \tag{3}$$

where **E** denotes a real Euclidean vector space (the space of translations) associated with E, and $T(S_0)$ is the lattice group of a Bravais lattice $S_0 \subset E$. This is equivalent to the assumption that the vector fields $\underline{E}_a$ $(a = 1, 2, 3)$ commute with each other:

$$[\underline{E}_a, \underline{E}_b] = 0 \tag{4}$$

The formula (3) implies that in this case the integral curve of the vector field $\underline{E}_a$ is transformed [with the mapping (1)] onto the *lattice line* (of the crystal latice S_0) with the directional vector $\overset{o}{\underline{E}}_a$ $(a = 1, 2, 3)$.

If the crystal lattice is distorted by the defects, then a mapping (1) with the property (3) does not exist, and in consequence

$$[\underline{E}_a, \underline{E}_b] = C^c_{ab}\underline{E}_c, \qquad C^c_{ab} \in C^\infty(\mathfrak{B}) \tag{5}$$

but the integral curves of the fields $\underline{E}_a$ can still be considered as the lattice lines of the crystal (Bilby, 1960).

Let us denote by Φ the teleparallelism on $\mathfrak{B}$ defined by the distribution (2) of the vector bases, and by ∇^Φ the covariant derivative, uniquely defined by the condition

$$\nabla^\Phi \underline{E}_a = 0, \qquad a = 1, 2, 3 \tag{6}$$

and let us denote by $\underline{S}[\Phi]$ the torsion tensor of the covariant derivative ∇^Φ. From (5) and (6) it follows that

$$\begin{gathered} \underline{S}[\Phi] = \underline{E}_a \otimes \tau^a \\ \tau^a = S^a_{bc} E^b \wedge E^c, \qquad S^a_{bc} = -\tfrac{1}{2} C^a_{bc} \end{gathered} \tag{7}$$

where $E^a(p) \in T^*_p(\mathfrak{B})$, $a = 1, 2, 3$, is the base dual to the base $E_p = E_{T(p)}$, and $\wedge$ denotes the exterior product. Because the integral curves of the fields $\underline{E}_a$ are the ∇^Φ-geodesics, and every ∇^Φ-geodesic is an integral curve of a certain Φ-parallel vector field $\underline{v}$, i.e., a field $\underline{v}$ such that

$$\nabla^\Phi \underline{v} = 0 \tag{8}$$

therefore the lattice lines of the distorted crystal structure can be identified with the ∇^Φ-geodesics.

In the case of a smooth distribution of dislocations in the body, the torsion tensor $\underline{S}[\Phi]$ is a measure of the density of this distribution, while the set of 2-forms $\tau = (\tau^a)$ can be considered as the infinitesimal counterpart of the so-called Burgers vector (Bilby, 1960). It is also known that dislocations have no influence on the local metric properties of the crystal structure of the body (Kröner, 1985). This can be described by assuming that the body $\mathfrak{B}$ is a Riemannian manifold with a metric tensor of the form

$$g(p) = g_{ab}E^a(p) \otimes E^b(p)$$
$$g_{ab} = \mathring{\underset{\sim}{E}}_a \cdot \mathring{\underset{\sim}{E}}_b = \text{const} \quad (\text{in } \mathbf{E}) \tag{9}$$

where the existence of a family $\{\underset{\sim}{\kappa}_p : T_p(\mathfrak{B}) \to \mathbf{E}, p \in \mathfrak{B}\}$ (in general non-integrable) of the local isomorphisms of property (3) was taken into account.

Because the ∇^Φ-geodesics are uniquely determined by the symmetric part of the covariant derivative ∇^Φ, i.e., by the covariant derivative ∇ of the form

$$\nabla = \nabla^\Phi - \underset{\sim}{S}[\Phi] \tag{10}$$

therefore this covariant derivative uniquely determines the geometry of lattice lines distortions caused by the occurrence of dislocations. In the base $E = (\underset{\sim}{E}_a, a = 1, 2, 3)$ the connection form ω_E of the covariant derivative ∇ has the representation:

$$\omega_E = \tfrac{1}{2}\underset{\sim}{C}_a \otimes E^a, \qquad \underset{\sim}{C}_a = \|C^b_{\ ac}\| \tag{11}$$

since in this base the connection form of the covariant derivative ∇^Φ disappears.

The description of dislocations in terms of the pair (Φ, ω_E) was proposed in Trzęsowski (1987, Section 1). According to that proposal, the teleparallelism Φ describes the breaking of translational symmetries of the lattice caused by the occurrence of dislocations in the body, whereas the connection form ω_E describes the type of distribution of these dislocations. We shall make this interpretation of ω_E clear with the use of an example of so-called closed teleparallelism.

A *closed teleparallelism* is defined by the condition

$$\nabla^\Phi \underset{\sim}{S}[\Phi] = 0 \tag{12}$$

which is equivalent to the condition

$$C^a_{\ bc} = \text{const} \tag{13}$$

A dislocation distribution fulfilling condition (12) will be called *uniformly dense*. In this case, according to formula (5), the vector fields $\underset{\sim}{E}_a$ span a three-dimensional, real Lie algebra of the Φ-parallel vector fields on $\mathfrak{B}$ [formula (8)]. This algebra is isomorphic with the matrix Lie algebra $\mathfrak{g} \subset gl(3)$, spanned by the matrices $\underset{\sim}{C}_a$ [formula (11)], since

$$[\underset{\sim}{C}_a, \underset{\sim}{C}_b] = C^c_{\ ab}\underset{\sim}{C}_c \tag{14}$$

Formulas (11), (13), and (14) mean that ω_E is a 1-form with the values in this Lie algebra $\mathfrak{g}$.

We can see then that if the teleparallelism Φ is closed, then all the possible types of lattice lines systems in the crystal with dislocations can be described by the well-known classification of three-dimensional real Lie algebras (e.g., Barut and Rączka, 1977). Thereby, we obtain the classification of types of crystal structure distortions by the uniformly dense distributions of dislocations, and in this sense we can speak about the *basic types of dislocation distributions*. The types of distribution of dislocations understood in this way were discussed in Trzęsowski (1987, Section 2). For example, if the fields $\underline{E}_a$ satisfy the additional condition

$$\nabla g = 0 \tag{15}$$

where g is defined by (9), then the Lie algebra $\mathfrak{g}$ is isomorphic with the Lie algebra $so(3)$ of the proper orthogonal group $SO(3)$. This is the case of so-called disclinations (Trzęsowski, 1987, Section 2.3).

The pair (Φ, g) defined by the conditions (9), (10), and (15) is called *consistent* (Wolf, 1972) and was discussed in Trzęsowski (1987, Section 2). Here we also use the notion of the *adjoint curvature tensor* $\underline{R} = \underline{R}[\Phi]$, introduced in that paper. This is the curvature tensor of the covariant derivative ∇; it disappears when the torsion tensor $\underline{S}[\varphi]$ disappears, and in the case of (12) it has the following properties (Schouten, 1954):

$$\begin{gathered} \nabla \underline{R}[\varphi] = 0 \\ R^{d}_{abc} = -\tfrac{1}{4} C^{p}_{ab} C^{d}_{pc} = \text{const} \end{gathered} \tag{16}$$

Let us consider as a geometrical object the pair (Φ, ω_E), where Φ is a closed teleparallelism. This pair describes some type, let us say $\mathfrak{g}$, of uniformly dense distribution of dislocations in the body. The question arises of how to construct a geometric object describing a distribution of dislocations that would not be uniformly dense, but that locally would be everywhere of type $\mathfrak{g}$. The procedure is known: we have to "gauge" the connection form ω_E. Construction of such a gauge transformation and a preliminary recognition of its basic properties is the aim of this paper.

2. THE GAUGE TRANSFORMATION

Let us consider the pair (Φ, ω_E), where Φ is a closed teleparallelism defined by a moving frame of the form

$$\begin{gathered} B(p) = B_p = (p, E_p) \\ E_p = (\underline{E}_a(p); \quad a = 1, 2, 3) \end{gathered} \tag{17}$$

and ω_E is the connection 1-form with values in the matrix Lie algebra $\mathfrak{g} \subset gl(3)$ defined by the formulas (5), (11), and (13). Let $G \subset GL(3)$ denote a connected matrix Lie group, the Lie algebra of which is the considered Lie algebra $\mathfrak{g}$ ($\dim \mathfrak{g} = 3$). We shall call the *distortion* the following mapping:

$$\underset{\sim}{S}: \quad \mathfrak{B} \to G, \qquad \underset{\sim}{S} = \|S^a_b\| \tag{18}$$

acting on vector bases $E = (E_p, p \in \mathfrak{B})$ according to the rule

$$E_p \to E_p \underset{\sim}{S}(p) = (\underset{\sim}{E}_a(p) S^a_b(p);\ b = 1, 2, 3) \tag{19}$$

Let us denote the set of all bases $e_p = (\underset{\sim}{e}_a(p);\ a = 1, 2, 3)$ of the tangent space $T_p(\mathfrak{B})$ as $B_p(\mathfrak{B})$, and let us introduce the following notations:

$$\begin{aligned} B_p(\mathfrak{B}, G) &= \{e_p = E_p \underset{\sim}{L}\colon \underset{\sim}{L} \in G\} \subset B_p(\mathfrak{B}) \\ B(\mathfrak{B}, G) &= \bigcup_{p \in \mathfrak{B}} B_p(\mathfrak{B}, G) \\ &\cong \{b_p = (p, e_p)\colon p \in \mathfrak{B},\ e_p \in B_p(\mathfrak{B}, G)\} \\ \pi\colon \quad B(\mathfrak{B}, G) &\to \mathfrak{B}, \qquad \pi(b_p) = p \end{aligned} \tag{20}$$

We can define the action of G on the set $B(\mathfrak{B}, G)$ to the right by the formula

$$\begin{aligned} b_p \underset{\sim}{L} &= (p, e_p) \underset{\sim}{L} = (p, e_p \underset{\sim}{L}) \\ e_p \underset{\sim}{L} &= (\underset{\sim}{e}_a(p) L^a_b), \qquad \underset{\sim}{L} = \|L^a_b\| \in G \end{aligned} \tag{21}$$

This action is effective,

$$b_p \underset{\sim}{L} = b_p \Leftrightarrow \underset{\sim}{L} = \underset{\sim}{I} = \|\delta^a_b\| \tag{22}$$

and transitive on every set $B_p(\mathfrak{B}, G)$,

$$\forall \underset{1}{e_p}, \underset{2}{e_p} \in B_p(\mathfrak{B}, G), \exists \underset{\sim}{L} \in G, \qquad \underset{2}{e_p} = \underset{1}{e_p} \underset{\sim}{L} \tag{23}$$

The definition of the set $B(\mathfrak{B}, G)$ implies that the moving frame (17) is the global cross section

$$B = \mathrm{id}_{\mathfrak{B}} \times E\colon \quad \mathfrak{B} \to B(\mathfrak{B}, G) \tag{24}$$

of this set, and defines the so-called trivialization φ_E:

$$\begin{aligned} \varphi_E &= \pi \times \hat{\varphi}_E\colon \quad B(\mathfrak{B}, G) \to \mathfrak{B} \times G \\ \hat{\varphi}_E(b_p) &= \underset{\sim}{S}(p) = \|S^a_b(p)\| \Leftrightarrow \underset{\sim}{e}_a(p) = \underset{\sim}{E}_b(p) S^b_a(p) \end{aligned} \tag{25}$$

The condition that φ_E be a diffeomorphism endows the set $B(\mathfrak{B}, G)$ with a structure of a connected smooth differentiable manifold. In this way the considered set $B(\mathfrak{B}, G)$ of G-equivalent bases has been endowed with the structure of the so-called principal fiber bundle (e.g., von Westenholz, 1978), with the distinguished smooth cross section (24).

We shall also deal with the vector bundle $\Psi(\mathfrak{B}, \mathfrak{g})$ of the form

$$\begin{gathered}\Psi(\mathfrak{B}, \mathfrak{g}) = \bigcup_{p\in\mathfrak{B}} \Psi(p) \\ \Psi(p) = L_R(T_p(\mathfrak{B}); \mathfrak{g}) \cong \mathfrak{g} \otimes T_p^*(\mathfrak{B}) \\ \pi_\Psi: \quad \Psi(\mathfrak{B}, \mathfrak{g}) \to \mathfrak{B}, \qquad \pi_\Psi(\underline{H} \otimes \alpha)_p = p\end{gathered} \tag{26}$$

where $T_p^*(\mathfrak{B})$ is the space dual to the tangent space $T_p(\mathfrak{B})$, and $\alpha \in T_p^*(\mathfrak{B})$. The differential structure on $\Psi(\mathfrak{B}, \mathfrak{g})$ can be defined with help of the condition that the mapping

$$\begin{gathered}\Psi_E: \quad \Psi(\mathfrak{B}, \mathfrak{g}) \to \mathfrak{B} \times (\mathfrak{g} \otimes R^3) \\ \Psi_E((\underline{H} \otimes \alpha)_p) = (p, \underline{H} \otimes \hat{\alpha}) \\ \hat{\alpha} = (\alpha_a(p);\ a \to 1, 2, 3), \qquad \alpha_a(p) = \alpha(E_a(p))\end{gathered} \tag{27}$$

is a diffeomorphism.

Consider a smooth mapping $\tilde{\omega}$ defined by

$$\begin{gathered}\tilde{\omega}: \quad B(\mathfrak{B}, G) \to \Psi(\mathfrak{B}, \mathfrak{g}) \\ \pi_\Psi \circ \tilde{\omega} = \pi\end{gathered} \tag{28}$$

The existence of the trivialization (25) allows reducing the mappings $\tilde{\omega}$ to the mappings ω defined by

$$\begin{gathered}\omega: \quad \mathfrak{B} \times G \to \Psi(\mathfrak{B}, \mathfrak{g}) \\ \tilde{\omega} = \varphi_E^* \omega = \omega \circ \varphi_E\end{gathered} \tag{29}$$

and also allows substituting the cross sections

$$b = \mathrm{id}_{\mathfrak{B}} \times e: \quad \mathfrak{B} \to B(\mathfrak{B}, G) \tag{30}$$

by the cross sections

$$b_E = b^* \varphi_E = \varphi_E \circ b = \mathrm{id}_{\mathfrak{B}} \times \underline{S}: \quad \mathfrak{B} \to \mathfrak{B} \times G \tag{31}$$

where $\underline{S}$ is a distortion [formula (18)] described by (25). Hence, the description of the mapping $\tilde{\omega}$, related to the cross section (30), has the following form (Mack, 1981):

$$\begin{gathered}(b^* \tilde{\omega})(p) = \tilde{\underline{\omega}}_a(b_p) \otimes E^a(p) \\ \tilde{\underline{\omega}}_a(b_p) = \underline{\omega}_a(p, \underline{S}(p)) + \underline{S}^{-1} \partial_a \underline{S}(p) \\ \partial_a \underline{S} = \|\partial_a S^b_c\| \\ (\partial_a S^b_c)(p) = \underline{E}_a(S^b_c)(p)\end{gathered} \tag{32}$$

where the global interpretation of the smooth vector fields $\underset{\sim}{E}_a$, i.e., their interpretation as differential operators $\underset{\sim}{E}_a : C^\infty(\mathfrak{B}) \to C^\infty(\mathfrak{B})$, is used (e.g., von Westenholz, 1978).

A mapping (28) such that the matrix function

$$\underset{\sim}{\omega}_a: \quad \mathfrak{B} \times G \to \mathfrak{g} \tag{33}$$

appearing in (32) satisfies the covariance condition

$$\forall \underset{\sim}{L} \in G \quad \underset{\sim}{\omega}_a(p, \underset{\sim}{S}\underset{\sim}{L}) = \underset{\sim}{L}^{-1}\underset{\sim}{\omega}_a(p, \underset{\sim}{S})\underset{\sim}{L} \tag{34}$$

assuring the independence of the matrix functions $\underset{\sim}{\omega}_a$ with respect to the choice of the cross section (30) is called the *Cartan-Ehrensmann connection form* (Mack, 1981). Then, the following holds:

$$\begin{gathered} \forall p \in \mathfrak{B} \quad \underset{\sim}{\omega}_a(p, \underset{\sim}{S}(p)) = \underset{\sim}{S}(p)^{-1}\underset{\sim}{\omega}_{E,a}(p)\underset{\sim}{S}(p) \\ \underset{\sim}{\omega}_{E,a}(p) = \underset{\sim}{\omega}_a(p, \underset{\sim}{I}) \end{gathered} \tag{35}$$

In particular, for the cross section (24),

$$\omega_E(p) = (B^*\tilde{\omega})(p) = \underset{\sim}{\omega}_{E,a}(p) \otimes E^a(p) \tag{36}$$

Formulas (32), (35), and (36) can also be written in the following form:

$$\sigma_{\underset{\sim}{S}^{-1}}(\omega_E) = \mathrm{Ad}(\underset{\sim}{S}^{-1})(\omega_E) + \underset{\sim}{S}^{-1}\, d\underset{\sim}{S} \tag{37}$$

Now we can "gauge" the connection form ω_E that describes the uniformly dense distribution of dislocations (see Introduction) by defining the family $\{\tilde{\omega}^b\}$ of the Cartan-Ehrensmann connection forms with the formula (Daniel and Viallet, 1980)

$$b^*\tilde{\omega}^b = \omega_E \tag{38}$$

where ω_E is described by equations (5), (11), and (13), and the index "b" runs through all the smooth cross sections (30). Then

$$B^*\tilde{\omega}^b = \sigma_{\underset{\sim}{S}}(\omega_E) = \mathrm{Ad}(\underset{\sim}{S})(\omega_E) + \underset{\sim}{S}\, d\underset{\sim}{S}^{-1} \tag{39}$$

Therefore, the distortion (18) defines the following gauge transformation:

$$\sigma_{\underset{\sim}{S}}: \quad (E, \omega_E) \to (E\underset{\sim}{S}, \sigma_{\underset{\sim}{S}}(\omega_E)) \tag{40}$$

The distribution of dislocations described by the connection form (39) will be called *locally uniformly dense.*

If (X^A) is the coordinate system on the manifold $\mathfrak{B}$, and

$$\begin{gathered} \underset{\sim}{e}_a = \underset{a}{e}^A\, \partial_A, \qquad \underset{\sim}{e} = \|\underset{a}{e}^A\| \in GL^+(3) \\ e^a = \overset{a}{e}_A\, dX^A, \qquad \underset{\sim}{e}^{-1} = \|\overset{a}{e}_A\| \\ \omega_E = \underset{\sim}{\omega}_{E'A} \otimes dX^A \end{gathered} \tag{41}$$

then, for $B_p=(p, \partial_{A|_p})$ and $\underset{\sim}{S}=\underset{\sim}{e}$, equation (39) takes the following form:

$$\sigma_{\underset{\sim}{e}}(\underset{\sim}{\omega}_{E'A})=\underset{\sim}{e}\partial_A\underset{\sim}{e}^{-1}+\underset{\sim}{e}\omega_{E'A}\underset{\sim}{e}^{-1}=\|\Gamma^B_{AC}\| \tag{42}$$

A formula having the form (42) was considered in Trzęsowski [1987, Section 2, equation (73)] as a formula describing the decomposition of a certain connection Γ. In that paper, the matrix function $\omega_{E,A}$ was called the *spinor connection.* In the general material space (Trzęsowski, 1987, Section 1.5) corresponding to this spinor connection the two transformation groups of (anholonomic) moving frames act nonhomogeneously: the matrix group $O_g(3)$, isomorphic to the orthogonal matrix group $O(3)$ and associated (in the base E) with the metric g having the form (9), and the matrix group G, which is being considered here. If (Φ, g) is a consistent pair, these groups coincide.

3. STRUCTURAL EQUATIONS

The connection form $\sigma_{\underset{\sim}{S}}(\omega_E)$ [formula (39)] can be represented in the following manner:

$$\begin{aligned}
\sigma_{\underset{\sim}{S}}(\omega_E)&=\underset{\sim}{C}'_a\otimes A[\underset{\sim}{S}]^a=\|\omega^a_b\|\\
A^a&=A[\underset{\sim}{S}]^a=A[\underset{\sim}{S}]^a_bE^b, \qquad \underset{\sim}{C}'_a=\tfrac{1}{2}\underset{\sim}{C}_a\\
\underset{\sim}{A}[\underset{\sim}{S}]&=\|A[\underset{\sim}{S}]^a_b\|: \quad \mathfrak{B}\to G\\
\omega^a_b&=C'^a_{cb}A[\underset{\sim}{S}]^c, \qquad C'^a_{cb}=\tfrac{1}{2}C^a_{cb}
\end{aligned} \tag{43}$$

where the gauge field $\underset{\sim}{A}[\underset{\sim}{S}]$ is defined by the condition

$$\underset{\sim}{C}'_bA[\underset{\sim}{S}]^b_a=\underset{\sim}{S}\underset{\sim}{C}'_a\underset{\sim}{S}^{-1}+\underset{\sim}{S}\,\partial_a\underset{\sim}{S}^{-1} \tag{44}$$

If $A=(\underset{\sim}{A}_a)$ is a vector base dual to the cobase $A^*=(A^a)$, and

$$\begin{aligned}
[\underset{\sim}{A}_a, \underset{\sim}{A}_b]&=\gamma^c_{ab}\underset{\sim}{A}_c, \qquad \gamma^c_{ab}\in C^\infty(\mathfrak{B})\\
[\underset{\sim}{E}_a, \underset{\sim}{E}_b]&=C^c_{ab}\underset{\sim}{E}_c, \qquad C^c_{ab}=\text{const}
\end{aligned} \tag{45}$$

then the Cartan structure equations have the form (Choquet-Bruhat, 1977; Rund, 1979)

$$\begin{aligned}
\tau^a&=dA^a+\omega^a_b\wedge A^b=S^a_{bc}A^b\wedge A^c\\
S^a_{bc}&=\tfrac{1}{2}(C^a_{bc}-\gamma^a_{bc})
\end{aligned} \tag{46}$$

for the torsion 2-form τ^a and

$$\Omega^a_b=d\omega^a_b+\omega^a_c\wedge\omega^c_b=\tfrac{1}{2}R^a_{cdb}A^c\wedge A^d \tag{47}$$

for the curvature 2-form Ω^a_b. Simultaneously, there holds

$$\Omega^a_b=C'^a_{cb}\Omega^c \tag{48}$$

where

$$\Omega^c = dA^c + \tfrac{1}{2}C'^c_{ab}A^a \wedge A^b = \tfrac{1}{2}F^c_{ab}A^a \wedge A^b$$
$$F^c_{ab} = \tfrac{1}{2}C^c_{ab} - \gamma^c_{ab}, \qquad R^a_{cdb} = \tfrac{1}{2}F^p_{cd}C^a_{pb} \tag{49}$$

Note that the above structure equations admit the existence of two kinds of fundamental states ("vacuum configurations") of the gauge fields $\underset{\sim}{A}[\underset{\sim}{S}]$. The condition of disappearing curvature

$$\Omega^a = 0 \tag{50}$$

defines the fundamental states associated with the structure constants C'^a_{bc}, i.e., the states described by the vector fields $\underset{\sim}{A}_a$, $a = 1, 2, 3$, which satisfy the equation

$$[\underset{\sim}{A}_a, \underset{\sim}{A}_b] = C'^c_{ab}\underset{\sim}{A}_c, \qquad \gamma^c_{ab} = \tfrac{1}{2}C^c_{ab} \tag{51}$$

The connection form $\sigma_{\underset{\sim}{S}}(\omega_E)$ corresponding to these fundamental states is defined by a certain closed teleparallelism Φ' with the torsion tensor $S'^a_{bc} = \frac{1}{4}C^a_{bc}$ [cf. (46)]. The condition of disappearing torsion

$$\tau^a = 0 \tag{52}$$

is equivalent to the condition

$$[\underset{\sim}{A}_a, \underset{\sim}{A}_b] = C^c_{ab}\underset{\sim}{A}_c, \qquad \gamma^c_{ab} = C^c_{ab} \tag{53}$$

and defines the fundamental states associated with the structure constants C^a_{bc}. These fundamental states correspond to the form $\sigma_{\underset{\sim}{S}}(\omega_E)$ of a certain symmetric connection, the curvature tensor of which has in the base $A = (\underset{\sim}{A}_a)$ the same components as the adjoint curvature tensor $\underset{\sim}{R}[\Phi]$ in the base $E = (\underset{\sim}{E}_a)$ [cf. formula (16)].

We can see, then, that the fundamental states corresponding to the condition (52) describe the same uniformly dense distribution of dislocations as the considered pair (Φ, ω_E). The distortions (18) corresponding to those states can be considered as the *local symmetries* of the discussed distribution of dislocations. From the formula (44) it follows that the gauge transformation is an automorphism of the Lie algebra $\mathfrak{g}$, i.e., that

$$\underset{\sim}{C}_b A[\underset{\sim}{S}]^b_a = \underset{\sim}{S}\underset{\sim}{C}_a\underset{\sim}{S}^{-1} \tag{54}$$

only when $\underset{\sim}{S} = \text{const}$. Then the Lie group G, considered as the group of automorphisms of the Lie algebra $\mathfrak{g}$, describes the *global symmetries* of the considered distribution of dislocations.

4. CONCLUSIONS AND REMARKS

The distortion $\underset{\sim}{S} : \mathfrak{B} \to G$ considered in this paper generalizes the notion of the distortion ideal crystal structure (cf. Trzęsowski, 1987, Section 1). In this generalization the uniformly dense distribution of dislocations of $\mathfrak{g}$ type (Introduction) is the counterpart of an ideal crystal structure, and the group G of the global symmetries of this distribution of dislocations (Section 3) is the counterpart of the symmetry group of the lattice. Consequently, the fundamental states of the gauge field here are not the states of the ideal crystal structure, but the states of some elementary distortion of this structure.

If the crystalline body with dislocations is not in the fundamental state, the gauge field describes the connection form having the nonzero forms of torsion (τ^a) as well as of curvature (Ω^a_b) [formulas (46)-(49)]. This can be interpreted as corresponding to the simultaneous occurrence of dislocations and point defects in the body (e.g., Günther and Żórawski, 1985; Trzęsowski, 1987, Section 2). It can be expected, then, that the connection form $\sigma_{\underline{S}}(\omega_E)$ [formulas (39), (43), and (44)] describes the secondary appearance of point defects in the body, e.g., due to intersection of the dislocation lines (cf. Günther and Żórawski, 1985).

REFERENCES

Barut, A. O., and Rączka, R. (1977). *Theory of Group Representations and Applications*, PWN, Warsaw.

Bilby, B. A. (1960). In *Progress in Solid Mechanics*, I. N. Sneddon and R. Hill, eds., p. 329, North-Holland, Amsterdam.

Choquet-Bruhat, Y. (1977). *Analysis, Manifolds and Physics*, North-Holland, Amsterdam.

Daniel, M., and Viallet, C. M. (1980). *Reviews of Modern Physics*, **52**, 175.

Günther, H., and Żórawski, M. (1985). *Annalen der Physik*, **42**, 41.

Kröner, E. (1985). *Dislocations and Properties of Real Materials*, Book 323, p. 67, Institute of Metals, London.

Mack, G. (1981). *Fortschritte der Physik*", **29** (4), 136.

Rund, H. (1979). *Tensor, N.S.*, **33**, 97.

Schouten, J. A. (1954). *Ricci-Calculus*, Springer-Verlag, Berlin.

Trzęsowski, A. (1987). *International Journal of Theoretical Physics*, **26**(4), 317.

Von Westenholz, C. (1978). *Differential Forms in Mathematical Physics*, North-Holland, Amsterdam.

Wolf, J. A. (1972). *Journal of Differential Geometry*, **6**, 317.